

Geometric Structures and Vector Distributions



TROMSØ
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STIFTELSE



UiT / THE ARCTIC UNIVERSITY
OF NORWAY

This 1-day conference on Tuesday 8 October 2024 is organized under the framework of the BFS/TFS project Pure Mathematics in Norway

Organizers: Omid Makhmali (UiT), Dennis The (UiT)

Conference location: UiT The Arctic University of Norway, FPARK B281

Registered Participants

Daniel Fox (UPM)	Marek Grochowski (UKSW)
Erlend Grong (UiB)	Boris Kruglikov (UiT)
Wojciech Kryński (IMPAN)	Omid Makhmali (UiT)
Irina Markina (UiB)	Eivind Schneider (UiT)
Wijnand Steneker (UiT)	Dennis The (UiT)
Luca Weigel (Marburg)	Henrik Winther (UiT)
Elias Knack (Marburg)	

Schedule

Talks are 40 minutes long followed by 5 minutes for questions

9:00 - 9:40	Boris Kruglikov	<i>Subconformal structures and dispersionless integrability</i>
9:45 - 10:05	Coffee break	
10:05 - 10:45	Irina Markina	<i>Local invariants and geometry of the sub-Laplacian on H-type foliations</i>
10:50 - 11:30	Marek Grochowski	<i>Existence and prolongation of infinitesimal isometries on special sub-Riemannian manifolds</i>
11:45 - 13:00	Lunch	
13:00 - 13:40	Erlend Grong	<i>Cartan geometry for ODEs and machine learning</i>
13:45 - 14:05	Coffee break	
14:05 - 14:45	Wojciech Kryński	<i>The Jacobi equation for path geometries</i>
14:50 - 15:30	Daniel Fox	<i>Contact path geometries revisited</i>

Titles and Abstracts

Daniel Fox (UPM)

Contact path geometries revisited

A classical path geometry on a manifold is a family of smoothly immersed curves in the manifold such that there is a unique curve in the family passing through a given point and tangent to a given direction. Locally any such family is equivalent to that generated by the images of the solutions of a system of second order ordinary differential equations. For example, the images of the geodesics of an affine connection, such as the straight lines in affine space, constitute such a family. Other more interesting examples will be described. It is interesting to consider smaller families of curves that are constrained in some way, for example by tangency to a distribution or a field of cones. The focus of the talk will be the case of families of curves tangent to a contact distribution, which are called contact path geometries. Their basic properties will be sketched, some examples will be given, and many questions will be raised. The "revisited" refers to the circumstance that the talk is based on work related to the speaker's thesis written more than twenty years ago.

Marek Grochowski (UKSW)

Existence and prolongation of infinitesimal isometries on special sub-Riemannian manifolds

I will speak about infinitesimal isometries of special sub-Riemannian manifolds (a contact and oriented sub-Riemannian manifold is called special if the Reeb vector field is an isometry).

The objective of my talk is to describe some conditions on such manifolds which allow one to construct, locally around a given point, infinitesimal isometries and then, possibly, to prolong them onto bigger domains. The mentioned conditions are related to the so-called i^* -regular and i -regular points, the notions introduced by Nomizu in 1960 in the Riemannian setting and slightly modified by the author.

Erlend Grong (UiB)

Cartan geometry for ODEs and machine learning.

We discuss how using the perspective of the frame bundle gives us to get a proper algebraic framework for numerical methods of solving ODEs on reductive homogeneous spaces. We will also look at how a frame bundle formulation help us design neural networks whose output is a tensor on a manifold. The application we have in mind is weather prediction. The research presented is part of joint works with Francesco Balerin, Nello Blaser, Hans Z. Munthe-Kaas and Jonatan Stava.

Boris Kruglikov (UiT)*Subconformal structures and dispersionless integrability*

I will report on the joint work with Omid Makhmali about subconformal geometry. The simplest example is a contact distribution with a sub-Riemannian structure defined up to scale. General structures of this type have invariants of order 0, however if we assume certain compatibility with the conformal symplectic structure on the distribution, the symbol is fixed. More precisely, we impose an integrable background constraint, which accidentally transforms this geometry into an instance of parabolic (A3,P13) type geometry (two real cases: CR and para-CR). However, in higher dimensions it is seldom parabolic but sometimes a Cartan geometry, which allows to compute differential invariants. For instance (4,6) distributions can come in 3 instances: elliptic, hyperbolic and parabolic. I will explain a relation of these structures with twistor theory and dispersionless integrability.

Wojciech Kryński (IMPAN)*The Jacobi equation for path geometries*

In this talk, I will present a formulation of the Jacobi equation for conjugate points within the framework of path geometries and control-affine systems.

Irina Markina (UiB)*Local invariants and geometry of the sub-Laplacian on H-type foliations*

Let (M, g) be a smooth, oriented, connected Riemannian manifold equipped with a Riemannian foliation with bundle-like complete metric g and totally geodesic leaves satisfying some additional symmetry conditions. The manifold is studied in the framework of sub-Riemannian geometry with bracket generating distribution transversal to the totally geodesic fibers. Equipping M with the Bott connection we find local invariants by studying the small-time asymptotic of the sub-Riemannian heat kernel. We obtain the first three terms in the asymptotic expansion of the Popp volume for the pull-back of small sub-Riemannian balls. We also address the question of local isometry of M as a sub-Riemannian manifold and its tangent group. This is the joint work with W. Bauer, A. Laaroussi (Leibnitz University of Hannover, Germany), S. Vega-Molino (University of Bergen, Norway)