MAT-3201 - syllabus

Dynamical Systems - 2007

(Lecturer Boris Kruglikov)

1. Basics of differential equations: Vector fields v. Cauchy theorem on existence and uniqueness of ODE $\dot{x} = v(x)$, $x \in U \subset \mathbb{R}^n$. Flows $\varphi(t, x)$, critical points and phase portraits.

2. Examples: ideal pendulum equation and its approximations. Newtonian (potential) systems.

3. General concept of a dynamical system: Discrete and continuous time dynamical systems. Connections between different type dynamical systems: Time-T maps, Poincaré map, Suspension.

4. Structure of solutions of linear ODE $\dot{x} = Ax$ and criterium of stability.

5. Lyapunov and asymptotic stability for nonlinear by the first approximation equations $\dot{x} = Ax + \alpha(x)$, $\|\alpha\| = o(\|x\|)$, $x \in \mathbb{R}^n$.

6. Floquet theory of linear ODE with periodic coefficients $\dot{x} = A_t x$, $A_{t+1} = A_t$. Stability in terms of the Floquet exponents.

7. Linearization of periodic systems $\dot{x} = A_t x$, $t \in S^1$, via time-dependent transformation.

8. Stability of systems depending on parameters. Parametric resonance.

9. Hamiltonian systems. Conservation of energy. Poisson brackets and their properties. Integrals.

10. Liouville theorem. Invariant tori for integrable Hamiltonian systems. Lissajous figures. Equilibrium points of Hamiltonian systems.

11. ω- and α-limit sets for dynamical systems. Their properties. Examples.

12. Qualitative theory of continuous time dynamical systems in dimension 2. Limit cycles. Poincare-Benedixon theorem.

13. Gradient systems and their ω- and α-limit sets. Equilibrium points.

14. Linearization via C^0-conjugation: Grobman-Hartman theorem. Hyperbolicity. Stable and unstable manifolds.

15. Equivalence of dynamical systems. Poincaré and Sternberg theorems on linearization. Resonances.

16. Topological characterizations of dynamical systems: minimality, topological transitivity, sensitivity to initial conditions. Chaos.

17. One dimensional dynamical systems: Rotation of the circle S^1. Expanding maps $E_m : I \to I$. Quadratic maps F_λ. Cantor set.

18. Higher-dimensional dynamical systems: Shifts on the tori $(\mathbb{T}^n, R_\gamma)$. Hyperbolic automorphisms of tori $(\mathbb{T}^n, A)$. Smale horseshoe.

References

[1] M.W. Hirsch, S. Smale, R.L. Devaney, *Differential equations, dynamical systems and introduction to chaos*, Elsevier, 2004.

[2] V.I. Arnold, *Ordinary differential equations*, Springer-Verlag, 2006 [old editions work as well].

[3] J. Palis, W. de Melo, *Geometric theory of dynamical systems. An introduction*, Springer-Verlag, 1982.

[4] R. Abraham, J.E. Marsden, *Foundations of mechanics*, Benjamin, 1978.