# Syllabus for MAT-3110 Differential Geometry I

A. Elementary topics (warm-up and additional questions):

1. Manifolds, submanifolds, orientation
2. Tangent space and vector fields
3. Cotangent space and 1-forms
4. Exterior algebra: Differential forms
5. De Rham differential *d*, relation to 3D vector calculus, and cohomology
6. Lie Derivative and commutation relations between $L_X$, $d$, $i_Y$.

B. Big topics (one to be chosen for the initial presentation):

1. Pfaffian systems (vector distributions)
    a. Integrable and non-integrable distributions [§ 4.1]
    b. Frobenius theorem [§ 4.2]
    c. Maurer-Cartan theory [§ 4.3]
2. Surfaces in the Euclidean 3-space:
    a. First, second fundamental form and Weingarten operator [§ 5.3]
    b. Gaussian and mean curvature [§ 5.4]
    c. Structure equations of surfaces and the Fundamental theorem [§ 5.2]
3. Abstract Riemannian manifolds:
    a. Riemannian metric [around]
    b. Levi-Civita connection and equation of geodesics [lectures]
    c. Curvature tensor and its components [§ 5.7]
4. Lie groups and Homogeneous Spaces:
    a. Lie groups and relations to Lie algebras [§ 6.1]
    b. Examples of Lie algebras [lectures]
    c. Transitive actions of Lie groups and homogeneous spaces [lectures and § 6.2]
5. Symplectic geometry:
    a. Symplectic vector spaces, isotropic and Lagrangian subspaces [§ 7.1]
    b. Symplectic form, Darboux theorem, Contangent bundle [§ 7.2]
    c. Poisson structure, Hamiltonian vector fields, integrals [§ 7.3]

C. Practice (easy computations to be able to present on the blackboard):

- Compute commutator of vector fields
- Compute exterior product of differential forms
- Hook vector field into a differential form
- Lie derivate a form along a vector field
- Compute de Rham differential of a differential form
- Verify (Frobenius) integrability of a vector distribution
- Compute Poisson bracket of two functions wrt the standard symplectic structure
- Be able to find Lie algebra of a (classical matrix) Lie group