Exercises 1.

1. Interpret geometrically the transformation $z \mapsto 2iz - 1$.

2. Draw the following set of points on the complex line $\mathbb{C}$:

   (a) Re$(z)> 2$.
   (b) $1 \leq |z| < 2$.
   (c) $1 <$Im$(z - i) < 2$.
   (d) $|z - 1| \leq 1$.
   (e) $|z - 1| < |z + 1|$.
   (f) Re$(z + 1) = |z - 1|$, use the characterizing property of parabola or rewrite the equation in terms of x, y.

3. Use de Moivre's theorem to show

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta, \cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

   Derive similar formulas for $\sin 5\theta, \cos 5\theta$.

4. Calculate all the roots $\sqrt{-3 \pm 4i}$.

5. Compare the set of roots:

   - $[(-1)^2]^{1/3}$ and $[(-1)^{1/3}]^2$.
   - $[(-1)^2]^{1/4}$ and $[(-1)^{1/4}]^2$.

6. Prove that if the polynomial $P(z) = a_n z^n + \ldots + a_1 z + a_0$ has real coefficients, $a_i \in \mathbb{R}$, and z_1 is a root, then $\bar{z}_1$ is also a root.

7. Find all the roots and the decomposition into polinomials of degree 1 of the following polynomials:

   - $P(z) = z^4 - 1$.
   - $P(z) = z^4 + 1$.
   - $P(z) = z^4 + 2z^3 + 3z^2 + 2z + 1$, one roots is $z_1 = \frac{-1+i\sqrt{3}}{2}$.
   - $P(z) = z^3 - 3z^2 + 3z - 2$, one root is $z_1 = 2$.

8. Prove that the mapping $z \mapsto w(z) = \frac{az+b}{cz+d}$ with $ad - bc \neq 0$ is a bijection between $\mathbb{C}\backslash\{-d/c\}$ and $\mathbb{C}\backslash\{a/c\}$. Show that the map transforms circles to circles.