MAT-3300: SYLLABUS

Polynomials with one variable

1. Extension fields, its degree, minimal polynomial, splitting field.

2. Algebraic extensions. Algebraic numbers. Algebraic closure.

3. Finite extensions of fields. Separable polynomials and separable extensions. Quadratic and cubic extensions.

4. Constructing with compass and ruler. Constructible numbers. Impossibility of some classical constructions in geometry.

5. Cyclomatic fields. Primitive roots. Cyclomatic polynomials.

6. Automorphism group of an extension. Galois extension and Galois group. Fixed sub-fields. Fundamental theorem of Galois theory.

7. Primitive elements. Simple extensions. Abelian & cyclic extensions. Application to compass and ruler construction of regular n-gons.

8. Galois group of a polynomial. Solvable groups. Theorem on solution of polynomial equations by radicals. Abel's insolvability theorem.

9. Infinite and transcendent extensions. Transcendence degree.

Polynomials with several variable

1. Noetherian ring. Hilbert's basis theorem. Polynomial rings.

2. Monomial orderings and their types. Gröbner basis and its existence. Buchsberger's algorithm.

3. Prime and radical ideals. Maximal ideals and their existence.

4. Affine algebraic sets $V = \mathcal{Z}(S)$ and vanishing ideals $I = \mathcal{I}(A)$. Coordinate ring of a variety. Its independence of an affine embedding.

5. Morphisms of varieties. Zariski topology. Zariski closure.

6. Projections onto affine subspaces. Noether's normalization lemma.

7. Hilbert's nullstellensatz. Algebraic geometry as correspondence between algebra and geometry: dictionary.

8. Field of rational functions on a variety. Dimension, singular points.

9. Local ring. Localization. Sheaf of local rings of a variety.

Practical knowledge

To be able to demonstrate ability to do with or without Maple:

1. Determine operations in a finite field extension $K|F$.

2. Compute cyclomatic and splitting field extensions given by an algebraic number or by a polynomial $f \in F[x]$.

3. Find abelian cyclic structure of the group $\mathrm{Gal}(\mathbb{Q}(\zeta_n))$.

4. The Galois group in low degree polynomails using (ir)reducibility and discriminant. Be able to interpret Maple output.

5. Compute reduced Gröbner basis of a polynomial ideal.

6. Check membership and equality for polynomial ideals. Compute the intersection and sum of polynomial ideals.

7. Verify if a given map is a morphism of varieties. Compute Zariski closure of the image of a morphism.

8. Determine if an ideal is prime. Compute radical of an ideal.

9. Compute the tangent space of a variety at a given point. Determine dimension of a variety.

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